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Multi-layered walls design to optimize building-plant interaction

Mario Ciampi, Francesco Leccese, Giuseppe Tuoni *

Dipartimento di Energetica "Lorenzo Poggi" – Università di Pisa – Via Diotisalvi, 2, 56126 Pisa, Italy

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Abstract

In this paper the building envelope suitable for making easier the purpose of the air-conditioning plant to keep the indoor air temperature constant against impulsive external temperature excitation is investigated. For this aim two different criteria can be assumed: the peak power minimization or the average time length maximization of the air-conditioning plant working step. Among all possible multi-layered walls, the symmetrical three-layered one, with the high heat capacity layer between two equal layers made of insulating material, turns out to be an excellent compromise fully satisfying the first criterion and, with good approximation, the second too. All the other usual walls, in particular the homogeneous single-layered ones, as well as the two-layered ones with the insulating layer disposed on the wall outer or inner face, and the three-layered ones with the insulating layer disposed in the mid-plane of the wall, have turned out to be distinctly worse.

Keywords: Building-plant interaction; Building envelope; Multi-layered walls; Impulsive external temperature excitation; Thermal comfort

1. Introduction

The determination of the most convenient sequence of resistive and capacitive layers in the building envelopes requires a careful specification of the parameters to be optimized. If such parameters vary, the optimal structure of the wall may remarkably vary too. In particular, in [1,2] the building envelope which, in case of intermittent heating, minimizes temperature fluctuations and the energy required by the plant are defined; in [3–8] walls are investigated in order to optimize the decrement factor and time lag under sinusoidal external thermal fluctuations.

A very interesting problem, relating to the building-plant interaction, is to determine the building envelope walls, which minimize the air-conditioning plant working in order to keep the indoor air temperature constant against outdoor air temperature variations with given temporal behaviour [9–15]. The solution, obviously, does not depend on the geometrical and thermal properties of the building interiors (e.g., floors and partitions), but only on the building envelope walls.

In the case of sinusoidal external temperature fluctuation, an approximate solution, in absence of air changes, is given in [9] and a fully developed analysis in [13], which we refer to for wide references.

In the case of impulsive external thermal excitation, two different criteria can essentially be followed to make easier the air-conditioning plant working: to minimize the peak power [10] or to maximize the average time length [13] of the plant working step. This subject is investigated in this paper, where multi-layered distributed-parameter walls are schematized as a sequence of essentially resistive layers alternating with essentially capacitive ones. First of all, the time response of the air-conditioning plant working step, with particular reference to its peak power and average time length, has been determined. Later, the results are applied to the simplest case of a lumped-parameter wall with purely resistive layers alternating with purely capacitive ones. The lumped-parameter model results to be so relevant as it represents the best limit solution.

In this paper a dimensionless parameter, called *coefficient of performance*, is introduced, which is able to quantify the deviation of a real multi-layered distributed-parameter wall from the limit behaviour of the optimal lumped-parameter model and, thus, to characterize the wall in its interaction with the plant [14–17].

The performance of different walls, made of materials commonly used in building, has been evaluated. In any case, the worst walls have turned out to be the homogeneous and the simple two-layered ones; optimal performance can

* Corresponding author.
E-mail addresses: m.ciampi@ing.unipi.it (M. Ciampi), f.leccese@ing.unipi.it (F. Leccese), g.tuoni@ing.unipi.it (G. Tuoni).

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Nomenclature

A, B	walls used in the calculations (see Tables 1, 2)	
c	wall heat capacity	$J \cdot m^{-2} \cdot K^{-1}$
c_p	specific heat at constant pressure	$J \cdot kg^{-1} \cdot K^{-1}$
E, F, G, H	elements of the wall transmission matrix	
H	five-layered lumped-parameter model (see Section 2)	
I	material with essentially resistive thermal properties	
ID	ideal three-layered lumped-parameter wall (see Section 8)	
I_e	intensity of the impulse	$K \cdot s$
K	five-layered lumped-parameter model (see Section 6)	
L	material with essentially capacitive thermal properties	
£	Laplace transform linear operator	
m_j	moment of j-order (see Section 3)	
M	transmission matrix of the wall	
p_1, p_2, p_3, p_4	dimensionless parameters, defined in Eq. (31)	
Q	heat flux due to the air-conditioning plant	$W \cdot m^{-2}$
r	wall thermal resistance	$m^2 \cdot K \cdot W^{-1}$
R_{int}	inner surface thermal resistance	$m^2 \cdot K \cdot W^{-1}$
R_{ext}	outer surface thermal resistance	$m^2 \cdot K \cdot W^{-1}$
s	$= j\omega$ complex variable ($j = \sqrt{-1}$)	
t	time	s
T_i	indoor air temperature	K
T_e	outdoor air temperature	K
T_w	wall inner surface temperature	K
x	L material dimensionless fraction on the wall inner face	
z	I material dimensionless fraction on the wall outer face	

Greek symbols

$\delta(t)$	Dirac delta function	
$\Delta\tau$	average time length of the air-conditioning plant working step	
ε	coefficient of performance, defined in Eq. (30)	
λ	thermal conductivity	$W \cdot m^{-1} \cdot K^{-1}$
$\phi(t)$	normalized response to the plant impulse	s^{-1}
ρ	density	$kg \cdot m^{-3}$
σ	dimensionless parameter, defined with, $= \omega rc$	
τ_0	thermal time constant of the wall	s
ω	frequency ($= 2\pi$/time period)	$rad \cdot s^{-1}$

Subscripts

A	referring to the wall A
B	referring to the wall B
ID	referring to the wall ID
ext	exterior
int	interior
I	resistive layer
L	capacitive layer
opt	optimal value

be obtained from symmetrical three-layered or five-layered walls.

2. Problem statement

We can take into account a wall composed of N homogeneous layers with a heat capacity C_k and a thermal resistance R_k ($k = 1, \ldots, N$) and two layers ($k = 0$ and $k = N + 1$), purely resistive ($C_0 = C_{N+1} = 0$), due to the wall inner and outer surface thermal resistances R_{int} and R_{ext} ($R_{N+1} = R_{int}$ and $R_0 = R_{ext}$). This wall can be represented by the following scheme:

$$\text{[interior]} \begin{pmatrix} R_{N+1} \\ 0 \end{pmatrix} \begin{pmatrix} R_N \\ C_N \end{pmatrix} \begin{pmatrix} R_{N-1} \\ C_{N-1} \end{pmatrix} \cdots \times \begin{pmatrix} R_2 \\ C_2 \end{pmatrix} \begin{pmatrix} R_1 \\ C_1 \end{pmatrix} \begin{pmatrix} R_0 \\ 0 \end{pmatrix} \text{[exterior]} \tag{1}$$

In any case, total thermal resistance r and heat capacity c are expressed by the following relations:

$$r = \sum_{i=0}^{N+1} R_i, \qquad c = \sum_{i=1}^{N} C_i$$

As is well known [3–5,18], in case of sinusoidal thermal fluctuation with angular frequency ω, the temperature T_i and the heat flux q_i on the interior side of the wall and the analogous quantities (T_e, q_e) concerning the outside are related as:

$$\begin{pmatrix} T_i \\ q_i \end{pmatrix} = \begin{pmatrix} E & F \\ G & H \end{pmatrix} \begin{pmatrix} T_e \\ q_e \end{pmatrix} = M \begin{pmatrix} T_e \\ q_e \end{pmatrix} \tag{2}$$

where E, F, G, H are the elements of the wall transmission matrix M of the external wall:

$$M = M(N+1) \cdot M(N) \cdot M(N-1) \cdots M(2) \cdot M(1) \cdot M(0)$$

In this relation $M(k)$ is the transmission matrix relating to the layer k ($k = 0, \ldots, N + 1$) and the factors are ordered as in the scheme (1). Without air change, the oscillating component of the heat flux Q [$W \cdot m^{-2}$] the air-conditioning plant has to provide to keep the indoor air temperature constant, coincides with q_i; from Eq. (2), with $T_i = 0$, we have [5,9,13]:

$$Q = -\frac{T_e}{F} \tag{3}$$

in which the wall transmission matrix M has been considered to have unitary determinant: $EH - FG = 1$. In general, T_e should be understood as sol–air temperature; under the same thermal external excitation, the bigger is $|F|$, the easier will be the air-conditioning plant working step. Moreover, with T_w wall inner surface temperature, we have: $Q = -T_w/R_{\text{int}}$ and it is possible to write $|T_w/T_e| = R_{\text{int}}/|F|$ as the wall inner surface decrement factor [5,8].

The matrix element F does not change under the effect of a specular reflection of the whole wall, which is obtained by inverting entirely the sequence of layers.

This property of the element F has a remarkable consequence. If, as it seems reasonable, only one optimal wall exists (with $|F|$ maximum) as resistance-capacity distribution varies, such a wall has, then, to be necessarily symmetrical by reflection. In fact, if the optimal wall were not symmetrical, it would be possible to obtain, by specular reflection, two walls with the same value of F; such walls would be distinct and both optimal. The optimal wall, if exists and is unique, is, therefore, necessarily symmetrical by reflection.

As a direct consequence, the simplest non-symmetrical walls such as, for example, the two-layered structures, with all the insulating material on the inner or outer surface of the wall, are to be excluded in the search for the optimal walls.

In case of sinusoidal thermal fluctuation, with angular frequency ω, the structures that maximize $|F|$ are the ideal lumped-parameter ones, made of a sequence of purely resistive and purely capacitive layers. Previous papers published by the same authors [5,9,13,14] show how the optimal sequence of resistive and capacitive layers turns out to be determined by the value of the parameter:

$$\sigma = \omega\tau_0$$

with $\tau_0 = rc$ the wall thermal time constant. The results can be resumed as follows.

By low values of σ ($\sigma < \sigma_1 \cong 18$) it is convenient to lump all capacity in only one layer with heat capacity c to be disposed in the mid-plane of the wall, and to lump resistances in two equal parts $r/2$ to be disposed in the outer layers of the wall. The optimal structure is made of three layers with one capacitive, indicated by the following scheme:

$$[\text{interior}]\left(\frac{r}{2}\right)(c)\left(\frac{r}{2}\right)[\text{exterior}] \tag{4}$$

For $\sigma_1 < \sigma < \sigma_2$ ($\sigma_2 \approx 42$) it is convenient to use a symmetrical, five-layered structure, with two capacitive layers, such as:

$$[\text{interior}]\left(\frac{r-r_1}{2}\right)\left(\frac{c}{2}\right)(r_1)\left(\frac{c}{2}\right)\left(\frac{r-r_1}{2}\right)[\text{exterior}] \tag{5}$$

In the scheme (5) r_1 is lower than $r/3$ [13]; the structure obtained from such scheme, considering in first approximation $r_1 \cong r/3$, will be, hereinafter, marked as H.

For higher values of σ the optimal distribution turns out to be still a symmetrical lumped-parameter one with a number of layers increasing with σ; by $\sigma \to \infty$ the optimal structure is the one of a homogeneous single-layered wall with resistive and capacitive parameters uniformly distributed.

In Mediterranean area the building envelope walls generally show values of $\sigma < \sigma_1$; only in case of particularly massive walls with remarkable thickness of insulating layer, values of $\sigma > \sigma_1$ are also possible. Anyway, in these cases, in almost all situations of practical interest, it turns out to be $\sigma < \sigma_2$. For example, in case of a 24 hour period of the thermal external fluctuation, for a homogeneous brick wall with a thickness of 25 cm and with density $\rho = 600\ \text{kg}\cdot\text{m}^{-3}$ we have $\sigma \cong 9$; for a concrete wall of 20 cm and with $\rho = 1600\ \text{kg}\cdot\text{m}^{-3}$ we have $\sigma \cong 6$. The presence of an insulating layer remarkably increases the value of σ: adding a polyurethane layer of 2 cm, the previous brick wall shows $\sigma \cong 14$, and adding the same insulating layer, the previous concrete wall shows $\sigma \cong 17$.

For a given wall thermal time constant, the value of σ is proportional to the frequency of the external thermal fluctuation and thus, for high frequencies, the corresponding value of σ might turn out to be high and will not be taken into account in this study, since the high frequency thermal fluctuations are transmitted by the wall with strong dampening and have a negligible influence on the indoor environment.

In case of impulsive external thermal excitation, the Eq. (3) has been written as a relation between the Fourier components and cannot be directly applied to determine the response to an impulsive thermal excitation; this relation can be easily extended to any transient thermal behaviour, by using Laplace transforms. In fact, by substituting in F the complex variable $s = j\omega$, the Eq. (3) can be understood as a relation between the Laplace transforms of the heat flux and the external air temperature; we, therefore, obtain:

$$\pounds(Q) = -\pounds(T_e)/F(s) \tag{6}$$

Into Eq. (6), the linear operator £, applied to a time function $Y(t)$, yields the Laplace transform [16]:

$$\pounds(Y) = \int_0^\infty e^{-st}Y(t)\,\mathrm{d}t \tag{7}$$

where s can assume any complex value.

In case of an impulsive excitation of the external temperature of any form, but of short time length compared to wall thermal time constant τ_0, we can write [5,10–14]:

$$T_e(t) = I_e\delta(t) \tag{8}$$

with $\delta(t)$ the Dirac delta function and I_e the “intensity” of the impulse [K·s].

From Eq. (8) we have $\pounds(T_e) = I_e$ and, from Eq. (6), we obtain [11]:

$$\pounds(Q) = -\frac{I_e}{F(s)} \tag{9}$$

In this case we can investigate the following subject: which is the distribution of resistance and capacity in the outer wall able to make easier the working step of the air-conditioning plant for given r, c and I_e. The plant working step is considered to be easier if heat flux is supplied uniformly distributed on a wide interval, as well as if its peak power value is small.

For this purpose, later on, two different criteria concerning the optimization of the resistance-capacity distribution in the wall will be taken into account:

- criterion of the maximum time length;
- criterion of the minimum peak power value.

3. Response length of the air-conditioning plant

In this section we aim at determining the whole heat power exchanged by the air-conditioning plant working step, the time lag and the length of such working. For this purpose it is not necessary to know the inverse Laplace transform of £(Q); if we know the Laplace transform £(Y) of the function $Y(t)$, it is possible to obtain directly the "moments" of $Y(t)$, that is to say the quantity:

$$m_j = \int_0^\infty t^j Y(t)\,\mathrm{d}t \quad \text{with } j = 0, 1, 2, \ldots$$

From Eq. (7), differentiating with respect to s, we obtain:

$$m_0 = \text{£}(Y)_{s=0} \qquad m_j = (-1)^j \left(\frac{\mathrm{d}^j \text{£}(Y)}{\mathrm{d}s^j}\right)_{s=0} \quad \text{for } j > 0 \tag{10}$$

If we identify the function $Y(t)$ with the time response $Q(t)$ of the air-conditioning plant, whose normalized trend is reported in Fig. 1 (see Section 4) the zeroth-order (m_0), the first-order (m_1) and the second-order (m_2) moments are particularly important. These moments show the physical meaning as follows.

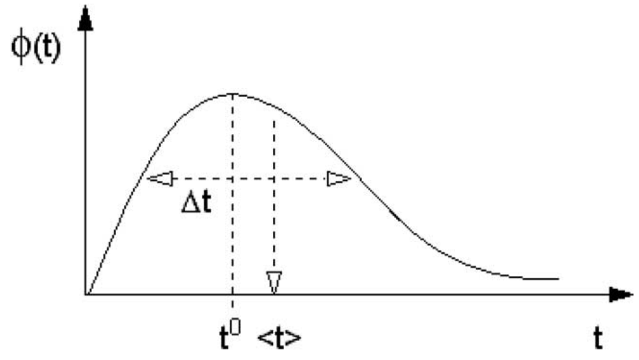


Fig. 1. Behaviour of the plant normalized response $\phi(t)$ varying with time: $\langle t\rangle$ temporal barycentre; Δt average time length; t^0 instant to which the function $\phi(t)$ reaches its maximum value.

The zeroth-order moment represents the energy supplied by the air-conditioning plant in response to the external thermal excitation:

$$m_0 = \int_0^\infty Q(t)\,\mathrm{d}t$$

From the first-order moment we obtain the quantity $\langle t\rangle$:

$$\langle t\rangle = \frac{\int_0^\infty t\,Q(t)\,\mathrm{d}t}{\int_0^\infty Q(t)\,\mathrm{d}t} = \frac{m_1}{m_0}$$

which can be read as the time barycentre of the power supplied by the plant, or rather as the mean value of t obtained by using as weighting function the quantity $Q(t)/m_0$; $\langle t\rangle$ quantifies the "time lag" between the external thermal excitation starting (at the instant $t = 0$) and the plant working step (see Fig. 1).

In the same way, from the second-order moment we obtain the quantity m_2/m_0, which can be read as the mean value of the square of the plant response time. It follows that the average time length Δt of the plant working step can be expressed, as standard deviation obtained using as weighting function $Q(t)/m_0$, by the following relation (see Fig. 1):

$$\Delta t^2 = \frac{\int_0^\infty (t - \langle t\rangle)^2 Q(t)\,\mathrm{d}t}{\int_0^\infty Q(t)\,\mathrm{d}t} = \frac{m_0 m_2 - m_1^2}{m_0^2}$$

In order to calculate the moments physically relevant of $Q(t)$, it is sufficient to know, from Eq. (10), a series expansion, truncated up to terms in s^2, of the Laplace transform (9); in other words the coefficients f_1 and f_2 of the expansion of the element F of the transmission matrix have to be known:

$$F(s) = r + f_1 s + f_2 s^2 + \mathrm{O}(s^3) \tag{11}$$

The lower-order moments of the heat power, from Eq. (10), result [5,11]:

$$m_0 = -I_e/r, \qquad m_1 = -f_1 I_e/r^2$$
$$m_2 = -2I_e\big(f_1^2/r - f_2\big)/r^2 \tag{12}$$

In particular, from the first of these relations it follows that the overall heat quantity, which must be supplied by the plant depends both on the intensity of the external temperature impulse itself and on the thermal resistance of the wall; it is not influenced by the wall heat capacity value and by the resistance-capacity distribution within the wall itself. On the contrary, the time lag and the time length, can be calculated as

$$\langle t\rangle = m_1/m_0 = f_1/r, \qquad \Delta t^2 = f_1^2/r^2 - 2f_2/r \tag{13}$$

Notice that the evaluation of $\langle t\rangle$ and Δt by Eq. (13) is exact and does not depend on the fact that the expansion convergence (11) is more or less rapid. The calculation of the coefficients f_1 and f_2 is reported in Appendix A.

By using the expressions of f_1 and f_2, see Eqs. (A.4)–(A.5), and the second of the Eq. (13), after some calculations, we find the following relation:

$$r^2\Delta t^2 = \sum_{i=1}^{n} C_i^2\left[\rho_i'^2(r-\rho_i')^2 - \frac{1}{3}R_i\left(r+\frac{R_i}{2}\right)\rho_i'(r-\rho_i') + R_i^2\left(\frac{R_i^2}{144}+\frac{r^2}{12}-\frac{rR_i}{60}\right)\right] + \sum_{i<k}^{n} C_iC_k\left[2\rho_i'^2(r-\rho_k')^2 + \frac{1}{6}R_i^2(r-\rho_k')^2 + \frac{1}{6}R_k^2\rho_i'^2 + \frac{1}{72}R_i^2R_k^2\right] \tag{14}$$

with ρ_k' thermal resistance preceding the kth layer mid-section, see Eq. (A.6).

The Eq. (14) provides the plant's response time length, under an impulsive external thermal excitation, in case of a real multi-layered wall.

4. Temporal behaviour of the air-conditioning plant response

In many cases it is important to determine the temporal behaviour of the heat flux $Q(t)$, which must be supplied by the plant: in particular, it is necessary if the maximum value of the heat power is to be calculated. In these cases, performing the inversion of the Laplace transform, from Eq. (9) it follows:

$$Q(t) = -\frac{I_e}{r}\cdot \pounds^{-1}\left(\frac{r}{F}\right) = -\frac{I_e}{r}\cdot\phi(t) \tag{15}$$

where, from Eq. (9), the function $\phi(t)$ is given by:

$$\phi(t) = \pounds^{-1}\left[\frac{r}{F}\right]$$

The function $\phi(t)$ is always positive; in Fig. 1 is shown the behaviour of ϕ as a function of the time.

For the first of the Eq. (12) it results:

$$\int_0^\infty \phi(t)\,\mathrm{d}t = 1 \tag{16}$$

while the quantity $\mathrm{Max}(\phi/r)$ measures the peak value of $Q(t)$ when $I_e = 1$ K·s. Owing to the proportionality between Q and ϕ, pointed out by Eq. (15), the quantities $\langle t\rangle$ and Δt, defined in Section 3, can be obviously visualized in the graph $\phi = \phi(t)$, Fig. 1.

The function $r/F(s)$ shows a discrete infinity of simple, generally distinct, poles all situated on the negative semi-axis of s, solutions of the equation $F(s) = 0$; these solutions, all negative, are here indicated by b_k and ordered in decreasing order with the index k ($\cdots < b_k < \cdots < b_2 < b_1 < 0$).

We, therefore, obtain the series [10,11]:

$$\phi(t) = \sum_{k=1\ldots\infty} B_k\cdot\exp(b_kt) \tag{17}$$

whose coefficients B_k, according to the method of residues, can be put in the following form:

$$B_k = r\Big/\left(\frac{\mathrm{d}F}{\mathrm{d}s}\right)_{s=b_k} \tag{18}$$

The series (17) converges rapidly except for very little time intervals; it, therefore, provides, together with Eq. (18), a useful method for the calculation of $Q(t)$; in all the cases examined in this paper it has been enough to take into account no more than seven terms in the series expansion (17).

Once the quantities (b_k, B_k) are known, it is possible to calculate the moments:

$$m_j = \int_0^\infty t^jQ(t)\,\mathrm{d}t = -\frac{I_e}{r}\int_0^\infty t^j\phi(t)\,\mathrm{d}t = -(j)!\cdot I_e\sum_{k=1\ldots\infty}\frac{B_k}{(-b_k)^{j+1}}$$

5. Maximum time length criterion

In order to investigate the maximum length criterion, it is convenient to use a simple lumped-parameter model. Other authors [19,20] describe the possibility to approximate the behaviour of building envelopes with lumped-parameter models. A structure made of a sequence of purely resistive and purely capacitive layers can approximate any distributed-parameter wall, either homogeneous or multi-layered. The use of an analysis based on a lumped-parameter model does not affect the generality of the results of this study.

We can subdivide the wall into n purely capacitive layers, with heat capacity c_s, and into $n+1$ purely resistive layers, with thermal resistance r_s; the wall structure, resulting with $2n+1$ layers, can be represented using the following scheme:

$$[\text{interior}](r_n)(c_n)(r_{n-1})\cdots(r_1)(c_1)(r_0)[\text{exterior}] \tag{19}$$

where the resistances r_0 and r_n are to be assumed as comprehensive of the wall inner and outer surface thermal ones. Total thermal resistance and heat capacity:

$$r = \sum_{s=0}^{n} r_s,\qquad c = \sum_{s=1}^{n} c_s$$

are kept constant.

Analogous relations concerning a lumped-parameter wall with the scheme (19) can be obtained, as a peculiar case, from the relations in Section 3. By using the expressions of f_1 and f_2, see Eq. (A.8), the second of the Eqs. (13) yields:

$$r^2\Delta t^2 = \sum_{i=1}^{n}\left[c_i\rho_i(r-\rho_i)\right]^2 + 2\sum_{i<k}^{n} c_ic_k\rho_i^2(r-\rho_k)^2 \tag{20}$$

where ρ_k represents the thermal resistance preceding thermal capacity c_k, see Eq. (A.7), $(r-\rho_k)$ means the resistance following capacity c_k and $(\rho_i - \rho_k)$ means the resistance between the two capacities c_i and c_k.

From Eq. (20) $\Delta\tau$ assumes its maximum value if, at the same time, f_1 results to be maximal and f_2 minimal. In order to have a high value of f_1 (with the same values of c and r), all capacities are to be disposed near the mid-plane, so as to increase as much as possible their weight $\rho_i(r - \rho_i)$: this is maximum for $\rho_i = r/2$. For this reason, if all capacity is lumped in only one layer, f_1 appears to be the maximum possible and results $f_1 = cr^2/4$; at the same time, f_2 is equal to zero and thus assumes the minimum possible value. In more formal terms, this condition can be realized by using a symmetrical structure with $n = 1$ such as the scheme (4), putting, for instance:

$$r_0 = r_1 = r/2, \qquad c_1 = c$$

$$r_k = 0, \qquad c_k = 0 \quad (\text{with } k > 1)$$

The average time length of the plant working step in optimal conditions results:

$$\Delta t_{\max} = \frac{1}{4}\tau_0 \tag{21}$$

In such conditions, also the time lag $\langle t \rangle$ results to be maximum and coincides just with the time length provided by Eq. (21). We can conclude that, in order to maximize the average time length $\Delta\tau$ of the heat power supplied by the plant, it is convenient to realize a symmetrical three-layered wall ($n = 1$), by disposing all heat capacity between two equal resistive layers. Such a wall (see Section 2) results to be optimal even in order to minimize the air-conditioning plant working, by low values of σ, in case of sinusoidal external temperature fluctuation.

6. Minimum peak power criterion

Also in this case it is convenient to use the lumped-parameter scheme (19), examining separately the walls characterized by various n. In this case every single layer in the scheme (19) is characterized by the matrixes

$$\begin{pmatrix} 1 & r_s \\ 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ sc_s & 1 \end{pmatrix}$$

according to whether it is a resistive or a capacitive one.

Obviously, the simplest interesting structure with the scheme (19) is the one characterized by $n = 1$; in this case the investigated wall results to be three-layered with the following scheme:

[interior]$(r_1)(c_1)(r_0)$[exterior]

The element F_1 of the wall transmission matrix with $n = 1$, results to be:

$$F_1 = r + r_0(r - r_0)cs = r(1 + a_1 s) \quad \text{with } a_1 = \frac{r_0}{r}(r - r_0)c \tag{22}$$

and the decay constant a_1 characterizing the plant response is obtained by solving the equation $F_1(s) = 0$.

Performing the inversion of Laplace transform indicated in Eq. (15) we have [11]:

$$\phi_1(t) = \frac{1}{a_1}\exp\left[-\frac{t}{a_1}\right] = \frac{r}{r_0(r - r_0)c}\exp\left[-\frac{rt}{r_0(r - r_0)c}\right]$$

which describes an exponential decay and shows its highest value $\text{Max}(\phi_1) = \phi_1(0) = 1/a_1$ (for $t = 0$). Varying r_0 ($0 < r_0 < r$), the decay constant a_1 assumes its highest value $a_1 = \tau_0/4$ when $r_0 = r/2$ (symmetrical wall); in these conditions the heat power required from the plant shows the minimum peak power value and we have:

$$\min_{(r_0)}\left[\mathop{\text{Max}}_{(t)}(\phi_1)\right] = \frac{4}{\tau_0} \tag{23}$$

Thus, when we restrict to $n = 1$, the minimum peak power criterion requires the choice of a symmetrical wall, made of a capacitive layer disposed between two equal resistive layers, which coincides with the scheme (4).

In case of a generic n the F_n is a n-degree polynomial, whose coefficients are positive and depend on the resistance and capacity elements of the scheme (19); if at least one of these elements is null, the polynomial assumes a degree smaller than n, showing that the investigated structure can be reduced to a lower degree. In this polynomial the zeroth-degree coefficient equals r; the highest-degree one is the product of all $2n + 1$ resistance and capacity elements composing the examined structure. All $F_n(s) = 0$ equation roots result to be real and negative and the function $F_n(s)$ can be explicated in the following form:

$$F_n(s) = r(1 + a_1 s)(1 + a_2 s)\cdots(1 + a_n s)$$

which generalizes the Eq. (22). The value of the decay constants a_k can be obtained from the roots s_k of the $F(s) = 0$ equation: $a_k = -1/s_k$; each of these time constants is positive and smaller than τ_0. These values will be assumed to be ordered in decreasing order: $a_1 \geqslant a_2 \geqslant a_3 \geqslant \cdots$. Supposing a_k as distinct values, the inverse Laplace transform ϕ_n of F_n can be expressed in the following form:

$$\phi_n(t) = \sum_{k=1\ldots n} A_k a_k^{n-2}\exp\left[-\frac{t}{a_k}\right] \quad \text{with}$$

$$A_k = \left[\prod_{\substack{i \neq k \\ i=1\ldots n}} (a_k - a_i)\right]^{-1} \tag{24}$$

By $n = 2$ the scheme (19) describes a five-layered wall:

[interior]$(r_2)(c_2)(r_1)(c_1)(r_0)$[exterior]

In this case the element F_2 of the transmission matrix M with $n = 2$, turns out to be a second-degree polynomial:

$$F_2 = r + \left[c_1 r_0(r_1 + r_2) + c_2(r_0 + r_1)r_2\right]\cdot s + r_0 c_1 r_1 c_2 r_2 \cdot s^2$$

with: $r_0 + r_1 + r_2 = r$ and $c_1 + c_2 = c$. From Eq. (24), by $n = 2$, we obtain:

$$\phi_2(t) = \frac{1}{a_1 - a_2}\left[\exp(-t/a_1) - \exp(-t/a_2)\right] \tag{25}$$

where the two decay constants are deduced from the solutions (s_1, s_2) of the second-degree equation: $F_2(s) = 0$, such as

$$a_1 = -1/s_1, \qquad a_2 = -1/s_2$$

The function $\phi(t)$ reaches its maximum value ϕ_2^0:

$$\phi_2^0 = \frac{1}{a_1 - a_2}\left[\left(\frac{a_2}{a_1}\right)^{a_1/(a_1-a_2)} - \left(\frac{a_2}{a_1}\right)^{a_2/(a_1-a_2)}\right] \tag{26}$$

at the instant t_2^0:

$$t_2^0 = \frac{a_1 a_2}{a_1 - a_2}\ln\left(\frac{a_1}{a_2}\right) \tag{27}$$

If we aim at determining the structure with $n = 2$ which minimizes ϕ_2^0, it is enough to just consider symmetrical structures, according to what is suggested by the symmetric property of the matrix element F, pointed out in Section 2. Considering a symmetrical structure of the type (5), with parameter r_1 $(0 < r_1 < r)$, the following expressions are obtained for the decay constants.

$$a_1 = \frac{1}{4}(r - r_1)c, \qquad a_2 = \frac{1}{4}(r - r_1)r_1\frac{c}{r} \tag{28}$$

Substituting Eq. (28) into Eqs. (25), (26) and (27) we obtain:

$$\phi_2(t) = \frac{4r}{c(r - r_1)^2}\left[\exp\left(-\frac{4t}{c(r - r_1)}\right) - \exp\left(-\frac{4rt}{cr_1(r - r_1)}\right)\right]$$

$$t_2^0 = -\frac{cr_1}{4}\ln\left(\frac{r_1}{r}\right)$$

$$\phi_2^0 = \frac{4r}{c(r - r_1)^2}\left[\left(\frac{r_1}{r}\right)^{r_1/(r-r_1)} - \left(\frac{r_1}{r}\right)^{r/(r-r_1)}\right]$$

As r_1 varies, ϕ_2^0 assumes its minimum value by $r_1 \cong 0.2032r$, where it results $\phi_2^0 \cong 3.344/\tau_0$. We can conclude that, for the case $n = 2$, the minimum peak power criterion involves that:

$$\min_{(r_1)}\left[\operatorname*{Max}_{(t)}(\phi_2)\right] \cong 3.344/\tau_0 \tag{29}$$

obtainable by a symmetrical five-layered wall with the scheme (5) with $r_1 \cong 0.2032r$; this scheme turns out to be, obviously, favourable compared to that relating to $n = 1$ for which the Eq. (23) is valid.

As r_1 varies, such minimum appears to be not very marked ("flat"); by $r_1 \to r$ we have $\phi_2^0 \to \infty$, while by $r_1 \to 0$ we have $\phi_2^0 = 4/\tau_0$, according to Eq. (23). E.g., by $r_1 = r/3$ we have $\phi_2^0 \cong 3.464/\tau_0$, while by $r_1 = r/2$ we have $\phi_2^0 = 4/\tau_0$. If we accept the criterion according to which an increase by 20% compared to the optimal value given by Eq. (29) can be admitted, the entire interval $0 \leqslant r_1 \leqslant r/2$ turns out to be acceptable.

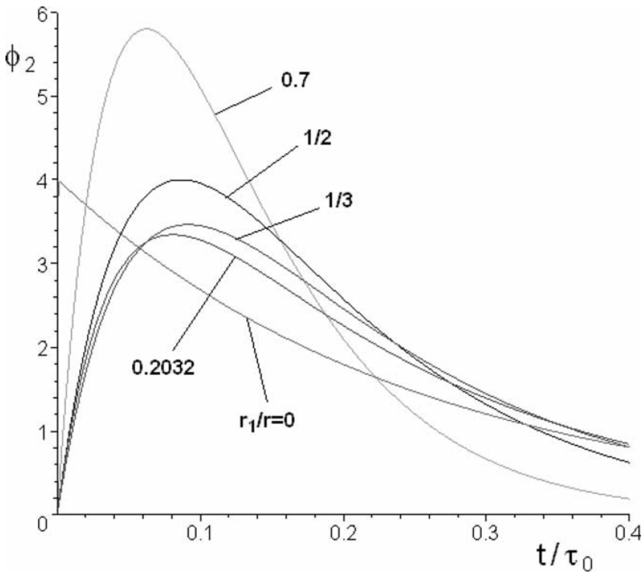


Fig. 2. The function ϕ_2 versus t/τ_0 in the symmetrical case $n = 2$ with different r_1/r values: $r_1/r = 0$, $r_1/r = 0.2032$, $r_1/r = 1/3$, $r_1/r = 1/2$ and $r_1/r = 0.7$.

In order to illustrate this situation, in Fig. 2 is shown, in the symmetrical case with $n = 2$, the behaviour of the function ϕ_2 versus t/τ_0, for: $r_1/r = 0$, $r_1/r = 0.2032$, $r_1/r = 1/3$, $r_1/r = 1/2$ and $r_1/r = 0.7$.

By $n = 3$ the scheme (19) describes a seven-layered wall:

[interior]$(r_3)(c_3)(r_2)(c_2)(r_1)(c_1)(r_0)$[exterior]

In this case the formulae result to be more difficult, so a numerical approach is to be preferred. As well as for the case $n = 2$, it is enough to restrict the analysis to symmetrical structures, with the resistance r_1 $(0 < r_1 < r/2)$ and the mid-centre plane capacity c_2 $(0 < c_2 < c)$ assumed as parameters. So we have:

$$r_0 = r_3 = \left(\frac{r}{2} - r_1\right), \qquad r_2 = r_1, \qquad c_1 = c_3 = \frac{c - c_2}{2}$$

On the basis of the values of such parameters the three time constants (a_1, a_2, a_3) can be calculated by solving the third-degree equation $F_3(s) = 0$, and the temporal response $\phi_3(t)$ can be determined by the Eq. (24). By solving the equation $(\mathrm{d}\phi_3/\mathrm{d}t) = 0$ the time t_3^0 can be individuated, so $\phi_3(t)$ results to be maximum and the maximum value ϕ_3^0 is obtained as: $\phi_3^0 = \phi_3(t_3^0)$. The numerical analysis shows that the minimum height peak conditions are achieved by $a_3 = 0$ and, therefore, coincide with those studied in the case of $n = 2$; so we have:

$$\min_{(r_1, c_2)}\left[\operatorname*{Max}_{(t)}(\phi_3)\right] \cong 3.344/\tau_0$$

These considerations can be generalized by $n > 3$ and lead to the conclusion that the plant peak power reaches its lowest value for a structure with $n = 2$ such as the previous one, and we have:

$$\min_{(n)}\Big[\operatorname*{Max}_{(t)}(\phi_n)\Big] \cong 3.344/\tau_0$$

We can, therefore, state that, in order to minimize the peak power required from the plant in its working step, the best possible wall with the scheme (19) results to be the symmetrical, five-layered one ($n = 2$) with $r_1/r = 0.2032$, obtained by disposing in the mid-plane resistive layer about 20% of the total thermal resistance r. This wall will be, hereinafter, marked as K.

7. The coefficient of performance

In the previous sections two different criteria, able to make easier the air-conditioning plant working, have been taken into account: to maximize the plant working step time length Δt, or to minimize the peak power of this working step. The optimal wall, determined with the first of these criteria, coincides with the optimal one obtained for low values of σ in case of sinusoidal external thermal fluctuations. This involves that the maximum time length criterion is to be preferred; on the other hand, if we compare different structures of real walls (with distributed parameters), the two criteria provide similar results.

For this reason we introduce a parameter defined as:

$$\varepsilon = 4\frac{\Delta t}{\tau_0} = \frac{4}{r\tau_0}\sqrt{f_1^2 - 2rf_2} \tag{30}$$

It is a *coefficient of performance* for the distribution of resistance and capacity within a wall and it can quantify the deviation of a real distributed-parameter wall from the behaviour of the lumped-parameter model.

For a multi-layered wall, the parameter ε can be calculated by Eq. (14) or, if the wall can be approximated to a lumped-parameter model, by Eq. (20). This parameter is a dimensionless quantity, able to assume numerical values from 0 to 1; in particular, it assumes its maximum value $\varepsilon = 1$ in the optimal case of a symmetrical three-layered wall with the scheme (4), as it clearly appears from Eqs. (21) and (30).

Neglecting the inner and outer surface thermal resistances we have: $\varepsilon = 0$ for a two-layered wall, with one layer purely resistive and the other purely capacitive, both in case of resistive layer on the inner face and on the outer face; $\varepsilon = (2/15)\sqrt{(10)} \cong 0.422$ for a homogeneous wall with continuous distribution of resistance and capacity.

The coefficient of performance is also related to the plant response under sinusoidal external thermal fluctuations with a low value of σ. This response, according to Eq. (3), is determined by the matrix element F. Under such a sinusoidal fluctuation, we have to put $s = j\omega = j\sigma/\tau_0$, so that the Eq. (11) provides for the $|F|$ the following expansion:

$$|F|^2 = r^2 + \left(f_1^2 - 2rf_2\right)\frac{\sigma^2}{\tau_0^2} + \mathrm{O}\left(\sigma^4\right)$$
$$= r^2\left[1 + \left(\frac{\varepsilon\sigma}{4}\right)^2\right] + \mathrm{O}\left(\sigma^4\right)$$

in which Eq. (30) has been used. This shows that ε can also be defined by the relation:

$$\varepsilon = \lim_{\sigma\to 0}(4/\sigma)\cdot\sqrt{|F/r|^2 - 1}$$

For this reason, in case of sinusoidal external thermal fluctuations and by low values of σ the search for the maximum value of the coefficient of performance ε allows to individuate the wall configuration which maximizes $|F|$ by reducing to the minimum the plant intervention (see Section 2). The coefficient of performance fully characterizes the behaviour of a wall, even in case of sinusoidal thermal fluctuations by low values of σ. In fact, in most cases of practical interest, the values of σ are low and just in case of massive and very well-insulated walls these values can result to be higher than σ_1 (see Section 2). The coefficient of performance turns out to be useful also in case of thermal excitations with complex spectrum, provided that there are not high frequency components with particularly remarkable intensity; in all these cases the maximization of ε represents very well the wall optimization [15–17].

8. Examples of walls commonly used in building

Multi-layered walls are generally used in the building envelope. Materials used in the realization of such walls are referable to two categories: the first type with essentially resistive thermal properties (I) and the other with essentially capacitive thermal properties (L).

Due to the practical importance of three-layered structures, it seems interesting to examine all possible three-layered two-component walls realized with given quantities of materials I and L. We can suppose of shifting a slice of material I inside a slab of material L, by introducing the dimensionless parameter x $(0 \leqslant x \leqslant 1)$ as the portion of material L on the wall inner face. We have the following scheme:

$$[\text{interior}](R_{\text{int}})[x\mathsf{L}][\mathsf{I}]\big[(1-x)\mathsf{L}\big](R_{\text{ext}})[\text{exterior}] \tag{31}$$

where the slice of material I shifts from the inside towards the outside by increasing values of the parameter x. By $x = 0$ the insulating material is disposed on the wall inner face (I–L walls), by $x = 0.5$ it is disposed in the mid-plane of the wall between two equal layers of material L (L–I–L walls) and by $x = 1$ it is disposed on the wall outer face (L–I walls).

In the same way, we can suppose of making material L float into material I, but it is better to make it move in the opposite direction, that is to say from the exterior inwards. In this case we can consider the following scheme:

$$[\text{interior}](R_{\text{int}})\big[(1-x)\mathsf{I}\big][\mathsf{L}][x\mathsf{I}](R_{\text{ext}})[\text{exterior}] \tag{32}$$

Table 1
Description of the walls thermal properties used in the calculations

Wall	Layers	d [cm]	ρ [kg·m^{-3}]	c_p [kJ·kg^{-1}·K^{-1}]	λ [W·m^{-1}·K^{-1}]	c [J·m^{-2}·K^{-1}]	$\tau_0 = rc$ [h]	$\sigma = \omega\tau_0$ (24 h period)
A	Polyurethane (I)	2.9	35	1.60	0.035	65000	28.9	7.56
	Cellular concrete (L)	12	600	0.88	0.20			
B	Polyurethane (I)	4.5	35	1.60	0.035	143000	63.6	16.7
	Concrete (L)	10	1600	0.88	0.70			

where the parameter x is the fraction of material I on the wall outer face.

By $x = 0$ and $x = 1$ the insulating material is disposed as in the previous cases (I–L and L–I), by $x = 0.5$ material L is disposed in the mid-plane of the wall between two equal layers of material I (I–L–I). The schemes (31) and (32) exhaust all possible cases of three-layered wall.

We have considered, as an example, the walls A and B described in Table 1 and characterized by the same value of total thermal resistance ($r = 1.60$ m^2·K·W^{-1}), by assuming for the wall inner and outer surface thermal resistances the reference values $R_{\text{int}} = 0.13$ and $R_{\text{ext}} = 0.04$ m^2·K·W^{-1} [EN ISO 6946/1996]. In Table 1 some important thermal parameters for both walls A and B are also shown.

It could be interesting to compare the behaviour of a wall with the schemes (31) and (32) to the behaviour of an ideal three-layered lumped-parameter wall with surface thermal resistances R_{int} and R_{ext}. In the same way, it could be interesting to compare the behaviour of a wall with the schemes (32) and (31) to the behaviour of three or five layered lumped-parameter walls.

A wall with the schene (32) in which material I is purely resistive and material L purely capacitive is defined as ID1; the wall turns out to be, therefore, a lumped-parameter, three-layer one, and can be represented by the scheme (19) with $n = 1$, where resistances have to be meant as comprehensive of that due to the surface thermal resistance. From Eq. (20) with $n = 1$ and from Eq. (30) we obtain:

$$\varepsilon = \frac{4}{r^2}\left[R_{\text{int}} + (1-x)(r - R_{\text{int}} - R_{\text{ext}})\right] \times \left[R_{\text{ext}} + x(r - R_{\text{int}} - R_{\text{ext}})\right] \tag{33}$$

From Eq. (33) it follows that the maximum value of ε ($\varepsilon = 1$) is obtained when $x = 0.554$.

Analogously, a wall with the scheme (31) in which material I is purely resistive and material L purely capacitive is defined as ID2; the wall turns out to be, therefore, a lumped-parameter, five-layer one, and can be represented by the scheme (19) with $n = 2$, where the extreme resistances coincide with the surface thermal resistances. From Eq. (20) with $n = 2$ and from Eq. (30) we obtain:

$$\varepsilon = \frac{4R_{\text{int}}R_{\text{ext}}}{r^2}\left[x^2\left(\frac{r - R_{\text{int}}}{R_{\text{ext}}}\right)^2 + (1-x)^2\left(\frac{r - R_{\text{ext}}}{R_{\text{int}}}\right)^2 + 2x(1-x)\right]^{1/2} \tag{34}$$

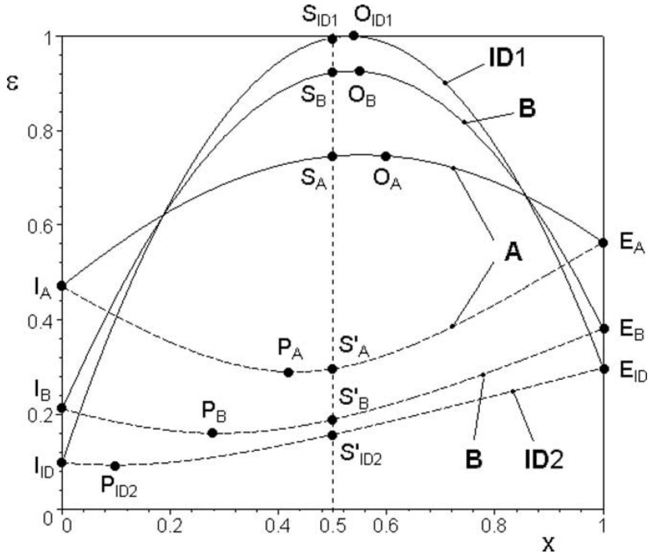


Fig. 3. Variation of the coefficient of performance as a function of x for the three-layered walls A and B with a scheme (31), dashed curves, and a scheme (32), continuous curves. The figure also shows the three-layered lumped-parameter wall, ID1 and ID2.

By Eq. (34) the maximum value of ε ($\varepsilon = 0.299$) is obtained when $x = 1$.

It should be noted that for the walls ID1 and ID2 the coefficient of performance does not depend on the wall heat capacity, but just on the total thermal resistance (r) and on the inner and outer surface thermal ones (R_{int} and R_{ext}).

In Fig. 3 the coefficient of performance is shown as a function of x for the structures with the schemes (31) and (32). As clearly shown in Fig. 3 all the walls A and B with a scheme (32) turn out to be better than the ones with a scheme (31). The wall ID1 represented by point O_{ID1} (with $x = 0.554$ and $\varepsilon = 1$) is the best one, while the wall ID2 represented by point P_{ID2} (with $x = 0.0958$ and $\varepsilon = 0.0927$) is the worst one.

The best real wall is the B-type one with a structure corresponding to the scheme (32) with $x \approx 0.55$ and $\varepsilon = \varepsilon_{\text{opt}} = 0.93$ (O_B), while for the wall A the best structure in the scheme (32) is the one with $x = 0.60$ to which corresponds $\varepsilon = \varepsilon_{\text{opt}} = 0.75$ (O_A).

The worst real wall is the B-type one with a structure corresponding to the scheme (32) with $x \approx 0.28$ and $\varepsilon = 0.16$ (P_B), while for the wall A the worst structure in the scheme (32) is the one with $x = 0.42$ to which corresponds $\varepsilon = 0.29$ (P_A).

It should be noted that for the wall A the coefficient of performance $(\varepsilon)_A$ varies from 0.75 (O_A) to 0.29 (P_A) and for the wall B the coefficient of performance $(\varepsilon)_B$ varies from 0.94 (O_B) to 0.16 (P_B), see Fig. 3. In other words, the walls having a high value of ε_{opt} show higher excursions in values of ε; such walls are better only if well designed.

The curves relating to the schemes (31) and (32) meet each other both when $x = 0$ and $x = 1$. When $x = 0$ the points I_A ($\varepsilon = 0.47$), I_B ($\varepsilon = 0.21$) and I_{ID} ($\varepsilon = 0.0975$), corresponding to I–L walls, are obtained. On the contrary, when $x = 1$ the points E_A ($\varepsilon = 0.56$), E_B ($\varepsilon = 0.38$) and E_{ID} ($\varepsilon = 0.2986$), corresponding to L–I walls are obtained.

Very important are the walls designed in a symmetrical way ($x = 0.5$) but with an asymmetry, from a thermal point of view, due to the inner and outer surface thermal resistances ($R_{int} \neq R_{ext}$). By $x = 0.5$ we have symmetrical, three-layered walls corresponding, in the case of the scheme (32), to the points S_{ID1} ($\varepsilon = 0.997$), S_B ($\varepsilon = 0.92$) and S_A ($\varepsilon = 0.74$), I–L–I walls and, in the case of the scheme (31), to the points S'_{ID2} ($\varepsilon = 0.157$), S'_B ($\varepsilon = 0.19$) and S'_A ($\varepsilon = 0.30$) (L–I–L walls).

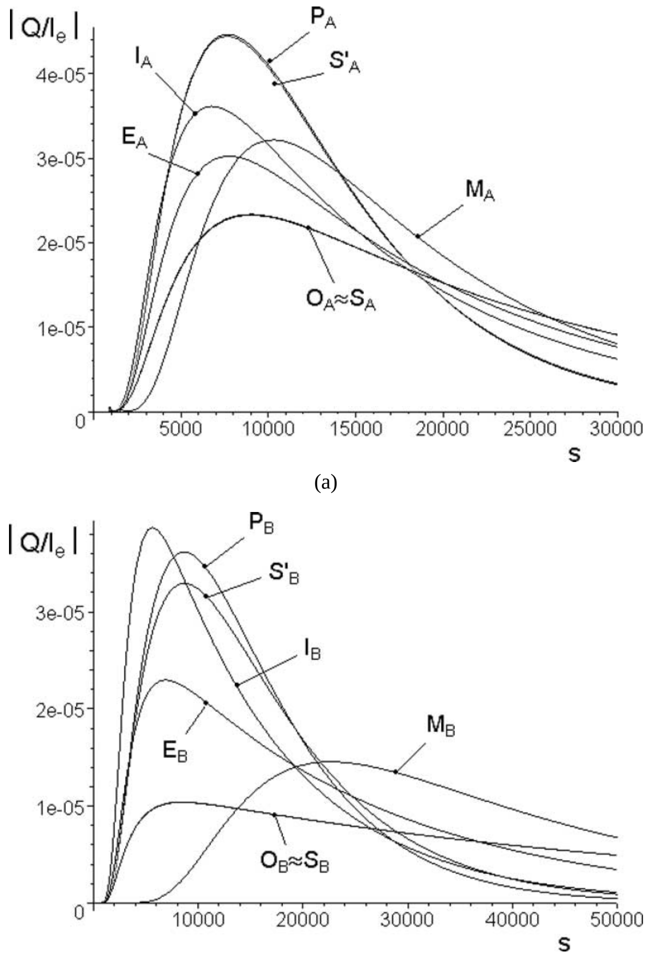
The structures O_{ID1} ($x \approx 0.55$, $\varepsilon = 1$), O_B ($x \approx 0.55$, $\varepsilon = 0.93$) and O_A ($x = 0.60$, $\varepsilon = 0.75$) represent the best realizations of the lumped-parameter model, see the scheme (4); such structures, as it results to be evident from the graph, are, for practical purposes, well approximated by the symmetrical ones represented by the points S_{ID2}, S_B, S_A.

A remarkably important case is represented by the homogeneous walls. Notice, on this subject, that materials with outstanding mechanical resistance and with low values of thermal conductivity have been recently introduced into the building practice. These materials (concrete made of expanded clay, pearlite and vermiculite, autoclaved and so on), produced in blocks or panels, make possible the realization of homogeneous walls with good mechanical and thermal properties. In case of a homogeneous wall with the reference values for the inner and outer surface thermal resistance and with the same total thermal resistance r as the walls A and B, we find that $\varepsilon = 0.47$, regardless of the value of the heat capacity (see Section 7). Homogeneous walls are then characterized by values of the coefficient of performance much lower than those, which can be obtained by using well-designed multi-layered walls, with the same r.

Some five-layered walls, obtainable by given thickness of the two materials I and L have been examined too. Among them a certain interest is presented by those, which approximate the following symmetrical lumped-parameter walls:

- H (see Section 2), represented by the scheme (5) with $r_1 \approx r/3$,
- K (see Section 6) represented by the scheme (5) with $r_1 = 0.2032r \approx r/5$.

The simplest realization is represented respectively by the following schemes:



(a)

(b)

Fig. 4. (a) Wall A: variation of $|Q/I_e|$ [$W \cdot m^{-2} \cdot K^{-1} \cdot s^{-1}$] as a function of time [s] for the walls corresponding to the points I_A, P_A, S'_A, E_A, S_A and O_A shown in Fig. 3; to have a comparison, the behaviour of a homogeneous wall, M_A, is also plotted. (b) Wall B: variation of $|Q/I_e|$ [$W \cdot m^{-2} \cdot K^{-1} \cdot s^{-1}$] as a function of time [s] for the walls corresponding to the points I_B, P_B, S'_B, E_B, S_B and O_B shown in Fig. 3; to have a comparison, the behaviour of a homogeneous wall, M_B, is also plotted.

$$[\text{interior}](R_{int})[I/3] \times [L/2][I/3][L/2][I/3](R_{ext})[\text{exterior}] \qquad (H_1)$$

$$[\text{interior}](R_{int})[2I/5] \times [L/2][I/5][L/2][2I/5](R_{ext})[\text{exterior}] \qquad (K_1)$$

For H_1 we find out: $\varepsilon = 0.60$ (A walls) and $\varepsilon = 0.68$ (B walls); for K_1: $\varepsilon = 0.66$ (A walls) and $\varepsilon = 0.78$ (B walls).

For all the structures being investigated the temporal trends of the plant response have been determined carrying out the calculations using the software MAPLE according to the method indicated in Section 4, i.e., solving the equation $F(s) = 0$ and using Eqs. (17), (18). In Fig. 4(a) relating to wall A, the variation of $|Q/I_e|$ [$W \cdot m^{-2} \cdot K^{-1} \cdot s^{-1}$] as a function of the time [s] is shown for the structures corresponding to the points I_A, P_A, S'_A, E_A, S_A and O_A of Fig. 3. In the same figure the variation of $|Q/I_e|$ for a homogeneous wall, M_A, with the reference values for inner and outer surface thermal resistances and with the same thermal resistance and capacity as wall A, is also shown. In Fig. 4(b), relating to wall B, the same quantity is shown for the structures corresponding to the points I_B, P_B, S'_B, E_B, S_B and O_B of Fig. 3 and for the homogeneous wall, M_B, with the reference values for inner and outer surface thermal resistances and with the same thermal resistance and capacity as wall B.

Since all the considered walls are characterized by the same thermal resistance value, in any case the same energy is required from the plant and therefore the area subtended by the curves results to be identical and, for the Eqs. (15), (16), equal to $1/r = 0.625$ [W·m^{-2}·K^{-1}].

In Fig. 4(a) and (b), the curves S_A and S_B result to be very close to the curves O_A and O_B; S'_A also is very near to the P_A, while you can notice a difference between S'_B and P_B, according to the above-remarked fact that the higher is the value of ε_{opt}, the more evident result to be the differences among the various walls. We can notice that the curves, with the exception of that relating to the wall I_B, are ordered in the direction of the decreasing coefficient of performance: as the coefficient of performance decreases, the peak power required from the plant during its intervention increases. The differences between the results of the two distinct criteria based on the impulse length and on its peak value tend to converge, and the lower is the value of ε_{opt}, the more narrow such convergence becomes.

In any case, the usual walls, in particular the homogeneous ones (M_A, M_B), the two-layered ones with the insulating material disposed on the outside (E_A, E_B) or with the insulating material disposed on the wall inner face (I_A, I_B), and the three-layered ones with the insulating material disposed in the mid-plane of the wall (S'_A, S'_B), result to be worse than the optimal ones (O_A, O_B).

In Fig. 5(a) and (b) the variation of $|Q/I_e|$ is plotted versus the time (s) for the five-layered structures (H_1, K_1); the behaviour of the homogeneous wall (M) and of the optimal three-layered one (O) is also shown. The Fig. 5(a) concerns the wall A and the Fig. 5(b) the wall B.

It is interesting to notice that, in the case of the wall B, the peak value of K_1 is a little lower than the one of O, according to the minimum peak criterion; on the contrary, for the wall A, provided with a lower ε_{opt}, the peak value of K_1 is slightly higher than that of O. In all the other cases plotted, the structures with higher ε show lower peaks. Therefore, also the investigation of the temporal trends of five-layered structures shows how the two criteria, based on the length and the peak value, give almost coincident directions for the wall A, and quite similar ones for the wall B; this confirms that the lower is ε_{opt}, the less the two criteria differ.

9. Conclusions

The problem concerning the determination of the building envelope structure able to facilitate the air-conditioning plant working step, in order to keep the indoor air temperature constant against impulsive external temperature excitations, can be investigated following two different optimization criteria: maximize the average time length or rather minimize the plant peak power.

On the basis of the first criterion, the optimal wall coincides with the symmetrical three-layered one, able to minimize the air-conditioning plant working step in case of sinusoidal external thermal fluctuations of low frequency. For such a wall, realized disposing all available heat capacity between two equal resistive layers, the average time length of the plant working step results to be equal to a quarter of the wall time constant.

On the basis of the second criterion, the optimal wall has turned out to be the symmetrical five-layered one with two capacitive layers and three resistive, realized by disposing in the mid-plane resistive layer the 20.32% of the wall total thermal resistance; in this case the peak power of the

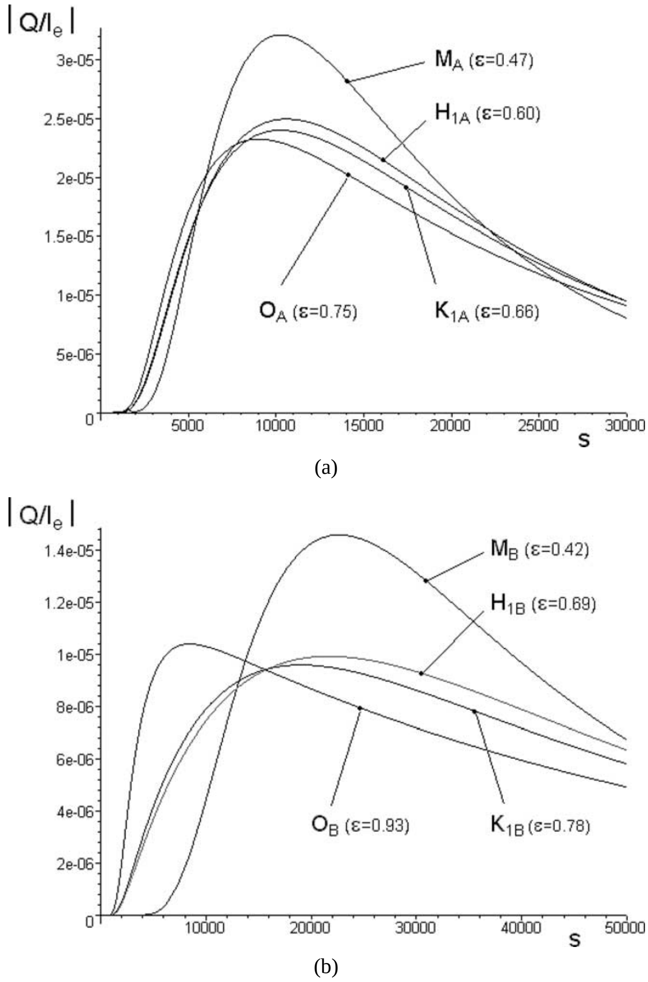


Fig. 5. (a) Wall A: variation of $|Q/I_e|$ [W·m^{-2}·K^{-1}] as a function of time [s] for the five-layered walls H_1, K_1; to have a comparison, the behaviour of the homogeneous wall, M, and of the optimal three-layered wall, O, is also plotted. (b) Wall B: variation of $|Q/I_e|$ [W·m^{-2}·K^{-1}] as a function of time [s] for the five-layered walls H_1, K_1; to have a comparison, the behaviour of the homogeneous wall, M, and of the optimal three-layered wall, O, is also plotted.

plant working step is inversely proportional to the wall heat capacity. For such a wall, if its mid-plane layer resistance is reduced, the peak power increases a little; so that the optimal wall, determined on the basis of the first criterion, shows a good performance on the basis of the second criterion as well.

Under the considered conditions, the optimal structure for an external wall can be stated to be the symmetrical three-layered one, obtained disposing all available heat capacity between two equal resistive layers. Notice that such a wall is very different from those of common use in building. All the usual walls, in particular the single-layered homogeneous ones, the two-layered ones with the insulating material disposed on the wall outer or inner face, as well as the three-layered ones with the insulating material disposed in the mid-plane of the wall, have resulted to be sensibly worse.

The previous analysis allows the introduction of a new parameter ε $(0 \leqslant \varepsilon \leqslant 1)$, called *coefficient of performance*, whose value quantifies the suitability of the distribution of resistive and capacitive layers in a multi-layer wall, within problems concerning thermal building-plant interaction.

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Appendix A

In a way similar to that used in Eq. (11), it is possible to expand, up to second-degree terms, the elements E, G and H of the transmission matrix:

$$\begin{pmatrix} E & F \\ G & H \end{pmatrix}$$

We obtain:

$$E = 1 + e_1 s + e_2 s^2 + \mathrm{O}(s^3) \tag{A.1}$$

$$G = cs + g_2 s^2 + \mathrm{O}(s^3) \tag{A.2}$$

$$H = 1 + h_1 s + h_2 s^2 + \mathrm{O}(s^3) \tag{A.3}$$

The elements of the transmission matrix for a generic layer k, with thermal resistance R_k and heat capacity C_k, can be given in hyperbolic functions form:

$$E_k = H_k = \cosh\bigl(\sqrt{R_k C_k s}\,\bigr) = 1 + \frac{1}{2} R_k C_k s + \frac{1}{24}(R_k C_k s)^2 + \mathrm{O}(s^3)$$

$$F_k = \sqrt{\frac{R_k}{C_k s}} \sinh\bigl(\sqrt{R_k C_k s}\,\bigr) = R_k\left[1 + \frac{1}{6} R_k C_k s + \frac{1}{120}(R_k C_k s)^2\right] + \mathrm{O}(s^3)$$

$$G_k = \sqrt{\frac{C_k s}{R_k}} \sinh\bigl(\sqrt{R_k C_k s}\,\bigr) = C_k s\left[1 + \frac{1}{6} R_k C_k s\right] + \mathrm{O}(s^3)$$

where series expansions truncated up to the second degree terms in s are also reported.

For the scheme (1) the layers with $k = 0$ and with $k = N + 1$ (wall inner and outer surface thermal resistances) are characterized by $E = H = 1$, $G = 0$ and F equal, respectively, to R_{ext} and to R_{int}. More generally, once the expansion of a matrix relating to a generic sequence of N homogeneous layers has been given, it is easy to find the expansion of the matrix (whose elements are marked with an overbar) for a sequence of $N+1$ layers. This can be obtained writing:

$$\begin{pmatrix} \overline{E} & \overline{F} \\ \overline{G} & \overline{H} \end{pmatrix} = \begin{pmatrix} E_{N+1} & F_{N+1} \\ G_{N+1} & H_{N+1} \end{pmatrix} \begin{pmatrix} E & F \\ G & H \end{pmatrix}$$

and neglecting terms whose degree is higher than the second. With some calculations we obtain the following expressions [5,11]:

$$\bar{e}_1 = e_1 + R_{N+1}\left(c + \frac{1}{2} C_{N+1}\right)$$

$$\bar{e}_2 = e_2 + \frac{1}{2} R_{N+1} C_{N+1} e_1 + R_{N+1} g_2 + \frac{1}{6} R_{N+1}^2 C_{N+1}\left(c + \frac{1}{4} C_{N+1}\right)$$

$$\bar{r} = r + R_{N+1}$$

$$\bar{f}_1 = f_1 + R_{N+1} h_1 + \frac{1}{2} R_{N+1} C_{N+1}\left(r + \frac{1}{3} R_{N+1}\right)$$

$$\bar{f}_2 = f_2 + \frac{1}{2} R_{N+1} C_{N+1} f_1 + R_{N+1} h_2 + \frac{1}{6} R_{N+1}^2 C_{N+1} h_1 + \frac{1}{24} R_{N+1}^2 C_{N+1}^2\left(r + \frac{1}{5} R_{N+1}\right)$$

$$\bar{c} = c + C_{N+1}$$

$$\bar{g}_2 = g_2 + C_{N+1} e_1 + \frac{1}{2} R_{N+1} C_{N+1}\left(c + \frac{1}{3} C_{N+1}\right)$$

$$\bar{h}_1 = h_1 + C_{N+1}\left(r + \frac{1}{2} R_{N+1}\right)$$

$$\bar{h}_2 = h_2 + \frac{1}{2} R_{N+1} C_{N+1} h_1 + C_{N+1} f_1 + \frac{1}{6} R_{N+1} C_{N+1}^2\left(r + \frac{1}{4} R_{N+1}\right)$$

which can be used as recursive relations for the calculation of all coefficients in Eqs. (11), (A.1)–(A.3). In particular, for the coefficients f_1 and f_2 of the expansion (11) for a wall with the scheme (1), we obtain:

$$f_1 = \sum_{k=1}^{N} C_k\left[\rho'_k\bigl(r - \rho'_k\bigr) - \frac{1}{12} R_k^2\right] \tag{A.4}$$

and

$$f_2 = \sum_{i<k}^{N} C_i C_k \left[\rho_i'(\rho_k' - \rho_i')(r - \rho_k') - \frac{1}{12} R_k^2 \rho_i' - \frac{1}{12} R_i^2 (r - \rho_k') \right] + \sum_{i=1}^{N} C_i^2 \left[\frac{1}{6} \rho_i' R_i (r - \rho_i') - \frac{1}{24} r R_i^2 + \frac{1}{120} R_i^3 \right] \quad \text{(A.5)}$$

with:

$$\rho_k' = R_1 + \cdots + R_{k-1} + \frac{1}{2} R_k = \sum_{j=1}^{k-1} R_j + \frac{1}{2} R_k \quad \text{(A.6)}$$

In the particular case of lumped-parameter model, see scheme (19), we should note that the capacitive layers only contribute to the terms of the sums in Eqs. (A.4), (A.5); moreover, of course, the terms containing $C_i R_i$ vanish. Furthermore, once a pure capacity c_k has been fixed, the quantity ρ_k' comes to the sum of resistances "preceding" this capacity in the scheme (19), and, therefore, identifies itself with the resistance ρ_k defined by:

$$\rho_k = \sum_{j=1}^{k-1} r_j \quad \text{(A.7)}$$

By using this approach we can deduce, from the relations in Section 3, the analogous ones valid for a lumped-parameter wall; in particular, from Eqs. (A.4), (A.5) we obtain:

$$f_1 = \sum_{i=1}^{n} c_i \rho_i (r - \rho_i)$$
$$f_2 = \sum_{i<k}^{n} c_i c_k \rho_i (\rho_k - \rho_i)(r - \rho_k) \quad \text{(A.8)}$$

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